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Softly broken $N = 4$ and E_8

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Abstract. We show that by adding supersymmetry breaking soft terms to $N = 4$ SYM, the E_8 gauge group can be spontaneously broken to SO_{10}, with the emergence of 4 conventional light generations of 16 plus mirror particles. However this is only achieved after fine tuning, with further tuning required if a Higgs sector is to appear at low energies.

1. Introduction

Recently it has been shown [1] that using a coset space dimensional reduction scheme on $N = 1$ super Yang-Mills (SYM) in $d = 10$, one could break both supersymmetry and the internal gauge group. When this group is taken to be E_8, one obtains a semi-realistic grand unification theory involving four massless generations (and 'mirror' generations) of SO_{10}.

In this paper we present the results of an attempt to obtain similar results using $N = 4$ SYM [2] in $d = 4$, with the E_8 gauge group. The scalar potential of this theory has flat directions—and moreover all the vacuum states are supersymmetric and thus degenerate among each other. One might hope that quantum corrections to the effective scalar potential would remove such ambiguities; however it has been shown that this is not possible in perturbation theory [5].

To remedy this situation we add to the $N = 4$ Lagrangian various dimension $\leqslant 3$ soft terms [3, 4] that explicitly break all four supersymmetries. One may then envisage constraining the parameters associated with such terms to single out the desired gauge breaking. It is well known [6] that $N = 4$ SYM has the property that it is finite to all orders in perturbation theory; the soft terms we add to the theory [3, 4] are all the dimension $\leqslant 3$ insertions that preserve this property.

In fact such $N = 4$ theories are members of a class of finite $N = 2$ theories [7, 8]. Although we know of no fundamental reason why these should be chosen above merely renormalisable field theories as candidates for grand unification, the Yukawa couplings of the former are very much more constrained than the latter, and this would appear more appealing.

It will turn out that the scale of E_8 and subsequent gauge breaking depend on the soft parameters—so that immediately in this scenario we have supersymmetry breaking at a high energy ($\geqslant 10^{15}$ GeV). This seems less attractive than the usual $N = 1$ phenomenological models (e.g. [9]) where supersymmetry is broken around M_w. Such models solve the so-called 'technical hierarchy problem'—that of maintaining $M_w \ll 10^{15}$ GeV through quantum corrections, in a very natural way. However, there are models where the scale of supersymmetry breaking is of the order of 10^{10} GeV, which then feeds down to the standard model, at $O(M_w)$. The reason why the scale of

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supersymmetry breaking, in our case, is so large is because the explicit SUSY breaking terms are also responsible for the breaking of the grand unified group, E_8.

Overlooking these difficulties for the moment, we will give an example of a chain of breaking involving the higher exceptional groups:

$$E_8 \to E_7 \to E_6 \to E_5(\sim SO_{10})$$

and show that E_7 and SO_{10} surviving symmetries correspond to minima of the scalar potential. We do not claim that this particular sequence of breaking has corresponding minima that are lower than another sequence, since this would require a full classification of the possible vacua of the scalar potential, which for E_8 would be difficult. But presumably, armed with such knowledge, one could arrange the soft terms so that a desired chain of breaking is rendered more favourable than any other—i.e. it corresponds to the lowest minima. In the following section we shall look closely at the details of tree level breaking in the scalar potential according to the above chain, and in particular at masses in the fermionic sector. Finally we make some conclusions as to the viability of the model in grand unified schemes.

2. Details of tree level breaking in the scalar potential

The component Lagrangian of $N=4$ SYM is given by [2]:

$$\begin{aligned}\mathscr{L} = \mathrm{Tr}\{&(-\tfrac{1}{4}G_{\mu\nu}G^{\mu\nu}) - \tfrac{1}{2}(\mathscr{D}_\mu A_i)^2 - \tfrac{1}{2}(\mathscr{D}_\mu B_i)^2 \\ &+\tfrac{1}{4}g^2[(A_i, A_j)^2 + (B_i, B_j)^2 + 2(A_i, B_j)^2] \\ &-\tfrac{1}{2}\bar{\lambda}_a \gamma^\mu \mathscr{D}_\mu \lambda_a + \tfrac{1}{2}g\bar{\lambda}_a[(\alpha^j_{ab}A_j + \beta^j_{ab}\gamma^5 B_j), \lambda_b]\}. \end{aligned} \tag{1}$$

The Lagrangian (1) has a $SU(2)_L \times SU(2)_R$ global symmetry with $a = 1$-4 labelling the $(2, 2)$ representation and $i, j = 1$-3 the $(1, 3)$ and $(3, 1)$ representations.

We wish to add finiteness preserving soft terms to the Lagrangian (1). The authors of reference [3] have listed all the possible soft terms (dimension $\leqslant 3$) that achieve this. They are combinations of the following in the notation of [3]:

$$\begin{array}{llll} A^2+B^2 & A^2-B^2 & \bar{\lambda}_a\lambda_b & \text{mass terms} \\ A(A^2+B^2) & A(A^2-3B^2) & & \text{cubic interactions.} \end{array} \tag{2}$$

These terms add an extra nine arbitrary parameters to (1)—four spinor and six scalar masses satisfying $\mathrm{STr}(\mathrm{mass})^2 = 0$.

Although we are putting in by hand the soft terms (2), one may conjecture that such terms could result from the spontaneous breaking of an $N=4$ supergravity theory, coupled to $N=4$ vector multiplets. In fact the generalised dimensional reduction [10] of $N=1$ supergravity plus SYM in $d=10$ yields just such a scenario in $d=4$ [11]. Two mass parameters are found to appear, along with all the independent soft terms (2). It could be that this is an indication of the type of breaking terms that can appear, by starting with the $d=4$ theory and employing a 'hidden sector' to break the local supersymmetry and then taking the $k \to 0$ limit. It is well known that in the analogous case of $N=1$ gauged matter coupled to $N=1$ supergravity [12] dimension $\leqslant 3$ explicit supersymmetry breaking terms appear in the $k \to 0$ limit.

Although a particular combination of the terms in (2) yield $N=1$ supersymmetric masses, we will not consider the addition of these to the theory; rather we will break all the degeneracies associated with surviving supersymmetries. The reason for this choice is that phenomenology rules out breaking of a grand unified gauge group in $N = 4$ SYM if, after breaking, there is at least one surviving supersymmetry (more of which will be said in the conclusion).

The first real difficulty one encounters in the details of breaking E_8 (or any group G for that matter) in our model is the lack of a 'hidden sector' in which E_8 could be broken, whilst protecting the low energy 'observable' sector of quarks and leptons from obtaining superheavy masses. This difficulty is a consequence of all the particles transforming under the same gauge group representations, because clearly one might expect that those Higgs bosons which give superheavy masses to 'unwanted' gauge bosons will also give superheavy masses to fermions having the same quantum numbers. But these are precisely the fermions whose mass we want to keep $O(M_w)$ since they correspond to the low energy sector. To give an example, if E_8 is broken down to a subgroup H say, then schematically we have

$$248 \rightarrow \mathrm{Ad}^{\mathrm{H}} + (\text{spinor rep})^{\mathrm{H}} + \text{others}^{\mathrm{H}}. \tag{3}$$

In (3), 248 is the adjoint of E_8, while the H label refers to the subgroup H. The gauge bosons in the spinor irreps of H are superheavy and since the fermions also transform under the 248, we might expect that only the masses of those in the Ad^{H} representation naturally keep their E_8 values, which we could make small. Already this looks troublesome because one can reiterate this pattern along the whole sequence of intermediate gauge groups, finally leading to a disastrous phenomenology. We emphasise that this is not a consequence of choosing E_8, but a property of any theory where all fermions are in the same representation as the gauge bosons.

Our only hope to combat the 'survival hypothesis' is that one may obtain massless fermions from the Yukawa couplings in (1), in the following ways.

(i) The diagonalised spinor mass matrix $\langle 0|A_i|(\alpha_i)_{ab} + B_i(\beta_i)_{ab}|0\rangle$ could have 'accidental' zero eigenvalues.

(ii) One may cancel mass terms in (i) against explicit spinor masses coming from the insertion of soft terms.

In $N = 4$ SYM (i) cannot yield zero modes in any other representation of H (the surviving gauge group) than the adjoint. This can be seen by taking the case where $\langle A_i\rangle \neq 0$ only (this is quite general because even if $\langle B_j\rangle \neq 0$, no cancellation with $\langle A_i\rangle$ can take place since $\bar{\lambda}\lambda$ and $\bar{\lambda}\gamma^5\lambda$ are independent), and realising that since $g^2\langle A^iA_i\rangle$ is the mass matrix of gauge bosons, its zero modes characterise the gauge bosons of H (all group indices implicit). So, the zero eigenvalues of $\langle A^iA_i\rangle$, which must be solutions of

$$\det(g^2\langle A^iA_i\rangle - M_\lambda^2) = 0$$

are precisely the solutions of $\det(g\langle A^i\rangle \pm M_\lambda) = 0$ where $g\langle A^i\rangle$ is the fermion mass matrix in 'group space'. Moreover since the 'generation' space part $(\alpha^i)_{ab}$ of this mass matrix is antisymmetric, 4×4 and real, all its eigenvalues are non-zero. Hence the zero modes of $\langle A^i(\alpha_i)\rangle$ characterise fermions in the adjoint representation of H.

Thus we will employ (ii) to fine tune massless fermions in representations R_α, of H, other than the adjoint. Choosing a convenient representation for the generators $(\alpha^i)_{ab}$ and taking $M^1, \ldots, M^4$ as the explicit spinor masses, we find that there are three massless modes in generation space if

$$q(\rho^-)^4 - (\rho')^2[(M^1 + M^2)(M^3 + M^4) + M^1M^2 + M^3M^4] + M^1M^2M^3M^4 = 0$$

$$\left(\sum_{i=1}^{4} M_i\right)3(\rho^-)^2 - M^1M^2(M^3+M^4) - (M^1+M^2)M^3M^4 = 0 \tag{4}$$

$$M^1M^2 + M^1(M^3+M^4) - 3(\rho')^2 = 0.$$

In equation (4) $\rho' = (g/2)\|\langle A^i\rangle\| N_\alpha$ for $i = 1, 2, 3$, with N_α a group factor depending on the particular representation R_α. Since we might expect ρ' to be a function of the mass scales introduced by the soft terms, equation (4) puts further constraints on these parameters.

If the adjoint of E_8 decomposes as

$$248 = \mathrm{Ad}^{\mathrm{H}} + \sum_\alpha R_\alpha^{\mathrm{H}} \tag{5}$$

then we can have at most $3 \times (R_\alpha^{\mathrm{H}})$ (for fixed α) of massless fermions. In (5), H refers to H the little group of $\langle A^i\rangle$ and α runs over representations other than the adjoint. The reason why only in one representation R_α^{H} can such cancellations take place, is because of the dependence of ρ' on R_α. The scalar potential plus soft terms [3, 4] is given by

$$\begin{aligned} V(\phi_i, \bar\phi^i) = \mathrm{Tr}\{&\tfrac{1}{2}|(\phi_i, \phi_j)\varepsilon_{ijk}|^2 g^2 + \tfrac{1}{2}|(\phi_i, \bar\phi^j)\varepsilon_{ijk}|^2 g^2 \\ &+ [\eta^{ijk}(\phi_i\phi_j\phi_k) + P_i^{jk}\bar\phi^i\phi_j\phi_k + \mathrm{HC}] \\ &+ \tfrac{1}{4}(M_j^{2i}\phi_i\bar\phi^j + N_{ij}^2\phi^i\phi^j + \mathrm{HC})\} \end{aligned} \tag{6}$$

where in equation (6)

$$\begin{aligned} &\phi_i = A_i + \mathrm{i}B_i = \phi_i^A Q_A \qquad [Q_A, Q_B] = \mathrm{i}f_{\mathrm{ABC}}Q_C \\ &A = 1\text{–}248 \qquad \eta^{ijk} = \tfrac{1}{3}\mathrm{i}gM_4\varepsilon^{ijk} \\ &P_i^{jk} = 2g\mathrm{i}M_{il}\varepsilon^{ljk} \qquad M_{il} = \mathrm{diag}(M_1, M_2, M_3). \end{aligned} \tag{7}$$

The requirement that $V(\phi_i, \bar\phi^i)$ be bounded from below implies

$$(M_{ij}^2 - N_{ij}^2) > 0. \tag{8}$$

The inequality (8) forbids any of the fields (A_i, B_i) from having a negative $(\text{mass})^2$, and so gauge breaking has to be driven by the cubic interactions in (6). We will take both A^2 and B^2 masses to be >0, so that V does not have any 'degenerate' minima. The first stage of breaking will be E_8 to E_7 via the maximal subgroup $E_7 \times SU_2$. Under this the decomposition of the adjoint of E_8 is

$$248 \to (133, 1) \oplus (56, 2) \oplus (1, 3). \tag{9}$$

From (9) we see that there are no singlets under $E_7 \times SU_2$; however, the $(1, 3)$ irrep preserves E_7, and we will use this to break E_8. To derive the potential involving $(1, 3)$ scalar fields requires the decomposition of the E_8 structure constants f_{ABC} with respect to $E_8 \times SU_2$—which can be obtained by decomposing the E_8 algebra (see [1]). Since we envisage E_7 singlets only, as picking up a non-zero VEV at this stage, it is sufficient to study the potential involving only these fields. This is given by

$$V(\phi_i^r, \bar\phi^{ir}) = \text{equation (6) with } f_{ABC} \text{ replaced by } \varepsilon_{rst}. \tag{10}$$

In (10) $r = 1\text{–}3$ labels the adjoint of SU_2. At this point we note that for the extrema equations

$$\delta V/\delta\phi^i = \delta V/\delta\bar\phi^i = 0 \tag{11}$$

to have solutions in which the vacuum expectation value $\langle \phi_i \rangle$ is non-zero requires that

$$[\phi_j, \phi_k]|_{\text{extrema}} \neq 0 \tag{12}$$

which means that we cannot diagonalise the fields ϕ_i (regarded as a matrix). Returning to the $E_8 \to E_7$ breaking, we see that it is natural that the fields ϕ_i align themselves in the three 'perpendicular' directions of the SU_2 adjoint $(1, 3)$. (By 'perpendicular' we mean non-commuting.) Exploiting the local SU_2 invariance and demanding real solutions to (11) we find

$$\langle \mathrm{Re}\, \phi_{i=1}^{r=1} \rangle = \langle \mathrm{Re}\, \phi_{i=2}^{r=3} \rangle = \langle \mathrm{Re}\, \phi_{i=3}^{r=2} \rangle = \rho \tag{13}$$

satisfies (11) and (12) if

$$M^{2ij} + N^{2ij} = M_+^2 \delta^{ij} \tag{14}$$

and ρ satisfies

$$g^2\rho^2 - 4g\rho\mu + \tfrac{1}{2}M_+^2 = 0 \tag{15}$$

where in equation (15) $\mu = (M_2 + M_3 - M_4 - M_1)$, and the reality of ρ implies $M_+^2 < 8\mu^2$. We note that condition (14) fixes two of the nine soft parameters.

One may check that (15) is sufficient to give masses of order ρ^2 to gauge bosons in $(56, 2)$ and $(1, 3)$ irreps in (9), leaving a theory invariant under E_7. To show that the solution (13) is a minimum of V, one has to check the positivity of all scalar $(\text{mass})^2$ after breaking occurs. From (6) we can derive the general formulae for A^2 and B^2 masses:

$$M_A^{2i\alpha j\beta} = (V^{,i\alpha}{}_{,j\beta} + V^{,j\beta}{}_{,i\alpha} + 2V^{,i\alpha,j\beta}) \tag{16}$$

$$M_B^{2i\alpha j\beta} = (V^{,i\alpha}{}_{,j\beta} + V^{,j\beta}{}_{,i\alpha} - 2V^{,i\alpha,j\beta}) \tag{17}$$

where $V^{,i\alpha} = \delta V/\delta\phi_{i\alpha}$, $V_{,i\alpha} = \delta V/\delta\bar{\phi}^{i\alpha}$ and in (16) and (17) $\alpha = 1 - \dim R_\alpha^{\mathrm{H}}$ labels the R_α representation of some subgroup H of E_8. The RHS of (16) and (17) is

$$\begin{aligned}(N^{2ij} \pm M^{2ij})\delta^{\alpha\beta} &+ 4\mathrm{i}P_l^{ij}(\phi^{Al})f_A^{\alpha\beta} + 2\mathrm{i}\eta^{ijk}f^{\alpha\beta A}\phi_{Ak} \\ &\pm 4\mathrm{i}(P_j^{il}\phi_l^B f_{\beta B}^{\alpha} + P_{jk}^{i}\phi^{ck}f_{c\alpha}^{\beta} + 3\eta^{ijk}f_A^{\alpha\beta}\phi_k^{*A}) \\ &+ \tfrac{1}{4}g^2 f^{A\alpha\gamma}\phi_{\gamma k}\varepsilon^{lik}\varepsilon_{lpj}f_{A\sigma}^{\beta}\phi^{*\sigma p} \pm \tfrac{1}{2}g^2(2f^{A\beta E}\phi_{Fk}\varepsilon^{lik}\varepsilon_{lpj}f_{AB\alpha}\phi^{pB} \\ &+ f_\alpha^{A\beta}\varepsilon^{lij}\varepsilon_{lpq}f_{ABc}\phi^{*Bp}\phi^{cq} + f_F^{A\beta}\phi_{Fk}\varepsilon^{pik}\varepsilon_{lpj}f_{A\alpha c}\phi^{cp}).\end{aligned} \tag{18}$$

In (18), $(\pm)$ refers to the A^2 and B^2 masses, respectively. For the particular case where $\mathrm{H} = E_7 \times SU_2$, the decomposition of f_{ABC} with respect to H, is given in [1]. Since the $(1, 3)$ scalars do not couple to those in $(133, 1)$, the latter all have positive $(\text{mass})^2$ by virtue of (8). We find that positivity of $(\text{mass})^2$ in the rest of the spectrum is not difficult to arrange, as long as the explicit scalar and spinor masses are of the same order of magnitude (which is natural, since finiteness imposes $\mathrm{STr}(\text{mass})^2 = 0$). As we shall see, in the breaking to SO_{10} we will not require explicit expressions for these masses.

Turning to the fermion mass corrections from $\langle (1, 3) \rangle$ we have the following Yukawa couplings:

$$\langle 1, 3 \rangle_{ab} \otimes [(56, 2)_a^{\bar{f}} \otimes (56, 2)_b^{f} \oplus (1, 3)_a^{\bar{f}} \otimes (1, 3)_b^{f}]. \tag{19}$$

In (19) the f refers to fermion representations. The details of the generation space and group representation indices are left out of (19). They can be obtained from the Lagrangian (1) by again making use of the decomposed structure constants f_{ABC}.

Earlier it was explained how it is natural to expect the fermions in the (133, 1) representation not to receive mass corrections from the (1, 3) Higgs fields (which is not apparent in (19)) but that the other fermions would require one to fine tune parameters in V in order that they receive no mass corrections. If we demanded that equations (4) hold we could get massless fermions in the 56 representations of E_7, but we should not expect them to remain so after further gauge breaking. Neither could one hope to cancel these further mass corrections, as they will be hierarchically larger than the $O(M_w)$ diagonal fermion masses obtained at E_7. We will therefore postpone imposing (4) until SO_{10} is reached, which will be the limit of E_8 breaking considered in this paper.

Having reached E_7 we will now break to SO_{10}, via the E_6 subgroup. Although it is possible that one could induce an E_6 invariant minimum of V, we will consider the direct breaking of E_7 to SO_{10} without any intermediate stages. Decomposing E_7 irreps with respect to E_6 and then SO_{10} we have

$$\begin{array}{ll}
E_7 & E_6 \times U_1 \\
133 & (78, 0)\oplus(27, -\frac{2}{3})\oplus(\overline{27}, \frac{2}{3})\oplus(1, 0) \\
56 & (27, \frac{1}{3})\oplus(\overline{27}, -\frac{1}{3})\oplus(1, 1)\oplus(1, -1) \\
E_6 & SO_{10}\times \tilde{U}_1 \\
27 & (10, \frac{1}{2})\oplus(16, \frac{1}{4})\oplus(1, \tilde{1}) \\
78 & (45, 0)\oplus(16, -\frac{3}{4})\oplus(\overline{16}, \frac{3}{4})\oplus(1, \tilde{0}).
\end{array} \tag{20}$$

In (20) there are a number of SO_{10} singlets, distinguished by their $\tilde{U}_1\times U_1\times SU_2^b$ quantum numbers, where SU_2^b is the broken SU_2 group factor of $E_7\times SU_2$. We label them

$$\begin{array}{ll}
[1, \tilde{0}, 0, 1]\equiv \tilde{S} & [1, \tilde{1}, -\frac{2}{3}, 1]\equiv P^1 \\
[1, \tilde{1}, -\frac{1}{3}, 2]\equiv \boldsymbol{P} & [1, \tilde{0}, 1, 2]\equiv \boldsymbol{T} \\
[1, \tilde{0}, 0, 1]\equiv S &
\end{array} \tag{21}$$

where in (21), $\boldsymbol{P}$ and $\boldsymbol{T}$ are SU_2 doublets, while P^1 is complex. $\tilde{S}$ and S are simply the $\tilde{U}_1$ and U_1 generators.

Each of the E_8 fields ϕ_i contains the singlets (21) but we may invoke a $\tilde{U}_1\times U_1$ transformation to rotate these SO_{10} singlets into a particular 'direction'. Since $[\phi_i, \phi_j]\neq 0$ in general, this rotation will have the effect of making these directions perpendicular for each value of i, and then we are guaranteed a global non-zero VEV by equation (11).

A judicious choice of $\tilde{U}_1\times U_1$ rotation would be

$$\mathrm{Re}\, \phi^s_{i=1}\to P^1 \qquad \mathrm{Re}\, \phi^s_{i=2}\to \bar{P}^1 \qquad \mathrm{Re}\, \phi^s_{i=3}\to \tilde{S} \tag{22}$$

where ϕ^s_i are the SO_{10} singlet fields in ϕ_i, because then only scalars in the 133 representation of E_7 pick up VEV. Previously we noted that the Higgs field that broke $E_8\to E_7$ did not couple to scalars in this representation, and so the task of minimising the potential of SO_{10} singlets (20) is considerably simplified. The relevant part of the scalar potential involving the fields in (22) is given by

$$\begin{aligned}
&\tfrac{1}{2}M_+^2(\tilde{S}^2+|P^1|^2)-4gM^{il}\varepsilon l^{jk}(\tilde{S}_iP^1_jP^{1'}_k+\text{cycle on } ijk) \\
&\qquad +\tfrac{2}{3}gM^4\varepsilon^{ijk}\tilde{S}_iP^1_jP^{1'}_k+\tfrac{1}{4}g^2(|\tilde{S}_iP^1_j\varepsilon^{ijk}|^2+|P^1_iP^{1'}_j\varepsilon^{ijk}|^2)
\end{aligned} \tag{23}$$

where in (23) $\tilde{S}_i$, P^1_j, $P^{1'}_k$ refer to $\mathrm{Re}\,\phi_i$ in the $\tilde{S}$, P^1, $\bar{P}^1$ irreps of (21) respectively. To obtain (23) one has to decompose the E_8 algebra with respect to $SO_{10}\times\tilde{U}_1\times U_1\times SU_2$ and find the commutation relations among the generators of (21).

Among the solutions of the extrema equations derived from (23) and which also satisfy the full E_8 extrema equations one finds

$$\begin{aligned} \mathrm{Re}(\phi^s_{i=1}) &= \mathrm{Re}(\phi^s_{i=2}) = (1/\sqrt{2}) M_+ (2a^2/M_+^2 - \tfrac{1}{2}g^2)^{-1/2} \\ \mathrm{Re}(\phi^s_{i=3}) &= a(2a^2/M_+^2 - \tfrac{1}{2}g^2)^{-1} \end{aligned} \tag{24}$$

the rest of the fields vanishing.

In (24) $a = 4g^2(M^1 - M^2 + M^3 + 2M^4)$. The interesting thing about the VEV in (24) is that one cannot have $\|\phi^s_i\| \ll$ scale of E_8 breaking because for this to happen $a/M_+^2 \gg 1$. But for the soft terms to maintain the finiteness of $N=4$ SYM, STr(mass)^2 must vanish. This implies that the largest positive (mass) from the scalar sector, M_+^2, must be of the same order as a^2. So E_8 and E_7 breaking occur at roughly the same energy scale. This close proximity of E_7 and SO_{10} invariant extrema makes the calculation of stability in the latter complicated, because of effects from the former. Nevertheless one can show that stability occurs, by using STr(mass)$^2 = 0$ to fix the ratios of highest explicit fermion to scalar masses. The only real constraints one has to impose are that corrections to the explicit scalar masses due to $E_8 \to E_7$ breaking do not change their order of magnitude, i.e. no accidental fine tuning must occur, and $\rho \gtrsim \tfrac{3}{2} M_+/g$ which is easily achieved in equation (15). Moreover one can maintain stability in both stages of breaking simultaneously. However, due to the close proximity in scales of breaking, it may be that stability in the latter stage is what one desires.

We may now study the Yukawa couplings involving the scalars (22). Since we have our eye on obtaining light or massless generations of fermions in the $16+\overline{16}$ irreps of SO_{10}, it will be sufficient to study only those couplings involving these fields.

From (20) we see that each 248 of E_8 gives rise to four $16+\overline{16}$ representations of SO_{10}, two from the (56, 2) and two from 133 when viewed from E_7. They are distinguished from each other by $\tilde{U}_1 \times U_1 \times SU_2$ quantum numbers. Denote the '16' fermions from the (56, 2) representations by $G_a^{(2)}$, and from the 133 by (G_{1a}, G_{2b}) where representation indices are suppressed. The Yukawa couplings of G are

$$\langle \tilde{S} \rangle_{ab} \{ \bar{G}_a^{(2)} \otimes G_b^{(2)} \oplus \bar{G}_{1a} \otimes G_{1b} \oplus \bar{G}_{2a} \otimes G_{2b} \} + (\langle P^1 \rangle_{ab} \{ \bar{G}_{1a} \otimes G_{2b} \} + \mathrm{HC}). \tag{25}$$

Because the mixing of G_{1a}, G_{2b} is complicated in (25), it is easier to try and obtain light fermions in $G_a^{(2)}$, which only couples to $\langle \tilde{S} \rangle_{ab}$. Remembering that these fermions also have an explicit mass matrix (corrected by $E_8 \to E_7$ breaking), $M_{ab} = \mathrm{diag}(M'_1, \ldots, M'_4)$, we find on computing the determinant of the mass matrix of $G_a^{(2)}$ in equation (25), that for two values of a, one can fine tune the mass away if

$$M'_1 M'_2 = M'_3 M'_4 = |(g/2)\langle \tilde{S} \rangle|^2. \tag{26}$$

In equation (26), $M'_i = M_i +$ corrections $O(\rho)$. The reason why this only happens for at most two and not three values of a (as stated in equation (5)) is because in the latter case we assumed that the VEV of the Higgs scalar A_i was in all three 'directions' of i. But now from (24) $\tilde{S}$ is seen to be non-zero in the $i=3$ direction only, and this *reduces* the number of modes we can fine tune to be massless.

Therefore, because the $G_a^{(2)}$ are doublets under the SU_2 factor of $E_7 \times SU_2$, we have the result that $4 \times 16 + \overline{16}$ massless fermions emerge at SO_{10}. We remark that using (24), equation (26) is a complicated constraint on the soft parameters (M_+^2, M_{ab}) which, along with the vanishing of STr(mass)^2, implies that of the nine initial parameters, five independent ones remain at SO_{10}. Therefore it should not be difficult to satisfy (26), and moreover this constraint does not affect the stability of the SO_{10} invariant minimum. The details of the $O(\rho)$ corrections to M_{ab} for various fermionic fields can be obtained from the determinant of the corresponding mass matrix after $E_8 \to E_7$.

3. Discussion

We have seen that, because of the severe constraint on the group representation content of $N=4$ SYM, the only way to obtain a light sector of fermions that correspond to conventional SO_{10} families is to fine tune the soft parameters (which are $O(\geqslant M_x)$) in the theory. This situation seems less attractive than ordinary SU_5 or SO_{10} grand unified theories, where the choice of fermionic representations prevents the future quarks and leptons from appearing in Yukawa terms involving the Higgs fields that have VEV $O(M_u)$ (M_u is the unification mass).

Neither can we avoid this fine tuning by explicitly breaking $N=4$ supersymmetry at low energies, i.e. by taking soft parameters to be of $O(M_w)$ because, although this would seem a more natural choice, it leads to a disastrous phenomenology if the scalars in the theory are taken to form a Higgs sector. This is because if $N=4$ SYM with a grand unified group G spontaneously breaks to G, say, at an energy scale M_u, with at least *one* supersymmetry remaining, then light ($O(M_w)$) fermions are degenerate with scalars lying in the same representations of G and having the same mass. Since these scalars are Higgs fields, then if their superpartners are charge $-\frac{1}{3}$ quarks, they will generate too rapid a proton decay [9]. It is precisely the choice of separate Higgs and matter supermultiplets in $N=1$ super GUT that avoids this danger, for then one can arrange for the supermultiplets containing the coloured Higgs scalars to acquire a mass $O(M_u)$.

One might also comment that the only light sector emerging from E_8 breaking (apart from gauge bosons) are the $16+\overline{16}$ families of SO_{10}. Clearly, further fine tuning is necessary if a Higgs sector is also to emerge which could then further break SO_{10}. Again this situation seems forced upon us by the constraints of $N=4$ SYM.

One might have hoped that the class of finite broken $N=2$ theories [7, 8], being less restricted in their representation content, could provide a more natural framework for a realistic finite model. However recent work [13] suggests that Higgs sector breaking may have to be ruled out for a large class of gauge groups on phenomenological grounds. Thus the relevance of known finite field theories in realistic models still seems uncertain.

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